Semi-Active Control of the Sway Dynamics for Elevator Ropes

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Abstract— **In this work we study the problem of rope sway dynamics control for elevator systems. We choose to actuate the system with a semi-active damper mounted on the top of the elevator car. We propose nonlinear controllers based on Lyapunov theory, to actuate the semi-active damper and stabilize the rope sway dynamics. We study the stability of the proposed controllers, and test their performances on a numerical example.**

I. Introduction

Modern elevators installed in high-rise buildings are required to travel fast and ensure comfort and safety for the passengers. Unfortunately, the dimensions of such high-rise buildings make them more susceptible to the impact of bad weather conditions. Indeed, when an external disturbances, like wind gust or earthquake, hits a building it can lead to large rope sway amplitude within the elevator shaft. Large amplitudes of rope sway might lead to important damages to the equipments that are installed in the elevator shaft and to the elevator shaft structure itself, without mentioning the potential danger caused for the elevator passengers. It is important then to be able to control the elevator system and damp out these ropes oscillations. However, due to cost constraints, it is preferable to be able to do so, with minimum actuation capabilities. Many investigations have been dedicated to the problem of modelling and control of elevator ropes [1], [2], [3], [4], [5], [6], [7]. In [4], a scaled model for high-rise, high-speed elevators was developed. The model was used to analyze the influence of the car motion profiles on the lateral vibrations of the elevator cables. The author in [1] proposed a nonlinear modal feedback to drive an actuator pulling on one end of the rope. The control performance was investigated by numerical tests. In [6], a simple model of a cable attached to an actuator at its free end was used to investigate the stiffening effect of the control force on the cable. An off-line energy analysis was used to tune an open-loop sinusoidal force applied to the cable. In [5], the authors proposed a novel idea to dissipate the transversal energy of an elevator rope. The authors used a passive damper attached between the car and the rope. Numerical analysis of the transverse motion average energy was conducted to find the optimal value of the damper coefficient. In [7], [8], [9], the present author studied the problem of an elevator system using a force actuator pulling on the ropes to add control tension to the ropes. The force actuator was controlled based on Lyapunov theory. Although, the sway damping performances obtained in [7], [8], [9] were satisfactory, we decided to investigate other forms of actuation and control. Indeed, in [7], [8], [9] we proposed a force control algorithm to modulate the ropes tensions, using an external force actuator, which was introduced at the bottom of the elevator shaft to pull on the ropes. However, retrofitting existing elevators with such a cumbersome actuator, can prove to be challenging and expensive. Instead, we propose to investigate here another actuation method, namely, using semi-active dampers mounted between the elevator car and the ropes. These actuators are less cumbersome than the typical force actuators and thus can be easier/cheaper to install. This type of actuation has already been proposed in [5]. The difference between the present work and [5], is that instead of using a static damper with constant damping coefficient tuned off-line, we use a semi-active damper, and use nonlinear control theory to design a feedback controller to compute online the desired time-varying damping coefficient that reduces the rope sway. We study the stability of the closed-loop dynamics, and show the performances of these controllers on a numerical example. One more noticeable difference with the work in [7], is that due to the semi-active actuation, the model of the actuated system is quite different from the one presented in [7]. Indeed, in [7] the model of the actuated system exhibited terms where the control variable was multiplied by the sway position variable only. In this work, the actuated model, exhibits terms where the control variable, i.e. the semi-active damper coefficient, is multiplied both by the sway position and

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the sway velocity variables, furthermore, the control variable term appears in the right hand side of the dynamical equations of the system, i.e. as part of an external disturbance on the system (refer to Section II). These differences in the model, make the controller design and analysis more challenging than in [7].

The paper is organized as follows: In Section II, we recall the model of the system when actuated with a semi-active damper mounted between the ropes and the elevator car. Next, in Section III, we present the main results of this work, namely, the nonlinear Lyapunov-based semi-active damper controllers, together with their stability analysis. Section IV is dedicated to some numerical results. Finally, we conclude the paper with a brief summary of the results in Section V.

Throughout the paper, $\mathbb{R}$, $\mathbb{R}_+$ denotes the set of real, and the set of nonnegative real numbers, respectively. For $x \in \mathbb{R}^N$ we define $|x| = \sqrt{x^T x}$, and we denote by $A_{ij},\ i = 1, ..., n,\ j = 1, ..., m$ the elements of the matrix A.

II. Elevator Rope Modelling

In this section we first recall the infinite dimension model, i.e partial differential equation (PDE), of a moving hoist cable, with non-homogenous boundary conditions. Secondly, to be able to reduce the PDE model to an ODE model using a Galerkin reduction method, we introduce a change of variables and re-write the first PDE model in a new coordinates, where the new PDE model has zero boundary conditions. Let us first enumerate the assumptions under which our model is valid: 1) The elevator ropes are modelled within the framework of string theory, 2) The elevator car is modelled as a point mass, 3) The vibration in the second lateral direction is not included, 4) The suspension of the car against its guide rails is assumed to be rigid, 5) The mass of the semi-active damper is considered to be negligible compared to the elevator car mass.

Under the previous assumption, following [3], [1] and [5], the general PDE model of an elevator rope, depicted on Figure 1, is given by

$$\begin{array}{l}\rho(\frac{\partial^2}{\partial t^2} + v^2(t)\frac{\partial^2}{\partial y^2} + 2v(t)\frac{\partial}{\partial y \partial t} + a\frac{\partial}{\partial y})u(y,t) \\ -\frac{\partial}{\partial y}T(y,t)\frac{\partial u(y,t)}{\partial y} + c_p\big(\frac{\partial}{\partial t} + v(t)\frac{\partial}{\partial y}\big)u(y,t) \\ +k_{dp}\big(\frac{\partial}{\partial t} + v(t)\frac{\partial}{\partial y}\big)u(y,t)\delta(y - l + l_{dp}) = 0\end{array} \tag{1}$$

where $u(y,t)$ is the lateral displacement of the rope. ρ is the mass of the rope per unit length. T is the tension

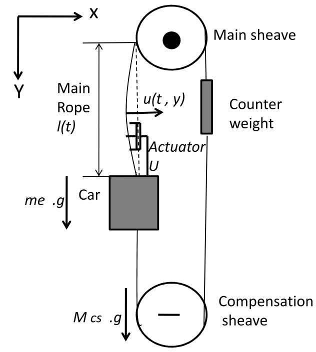

Fig. 1. Schematic representation of an elevator shaft showing the different variables used in the model

in the rope, which varies depending on which rope in the elevator system we are modelling, i.e. main rope, compensation rope, etc. c_p is the damping coefficient of the rope per unit length. $v = \frac{\partial l(t)}{\partial t}$ is the elevator rope velocity, where $l:\ \mathbb{R} \to \mathbb{R}$ is a function (at least C^2) modelling the time-varying rope length. $a = \frac{\partial^2 l(t)}{\partial t^2}$ is the elevator rope acceleration. The last term in the righthand-side of (1) has been added to model the semi-active linear damper effect on the rope at the contact point $\delta(y - l + l_{dp})$, where δ is the Dirac impulse function, k_{dp} is the damping coefficient (the control variable) and l_{dp} is the distance between the car-top and the point of attach of the damper to the controlled rope, e.g. [5].

The PDE (1) is associated with the following two boundary conditions:

$$\begin{array}{l}u(0,t) = f_1(t) \\ u(l(t),t) = f_2(t)\end{array} \tag{2}$$

where $f_1(t)$ is the time varying disturbance acting on the rope at the level of the machine room, due to external disturbances, e.g. wind gust. $f_2(t)$ is the time varying disturbance acting at the level of the car, due to external disturbances. In this work we assume that the two boundary disturbances acting on the rope are related via the relation:

$$f_2(t) = f_1(t)sin\big(\frac{\pi(H-l)}{2H}\big),\ H \in \mathbb{R} \tag{3}$$

where H is the height of the building. This expression is an approximation of the propagation of the boundary disturbance f_1 along the building structure, based on the length l, it leads to $f_2 = f_1$ for a length 0 (which

is expected), and a decreasing force along the building until is vanishes at $l = H$, $f_2 = 0$ (which makes sense, since the effect of any disturbance f_1, for example wind gusts, is expected to vanish at the bottom of the building). As we mentioned earlier the tension of the rope $T(y)$ depends on the type of the rope that we are dealing with. In the sequel, we concentrate on the main rope of the elevator, the remaining ropes are modelled using the same steps by simply changing the rope tension expression.

For the case of the main rope, the tension is given by

$$T(y,t) = (m_e + \rho(l(t) - y))(g - a(t)) + 0.5M_{cs}g \quad (4)$$

where g is the standard gravity constant, m_e, M_{cs} are the mass of the car and the compensating sheave, respectively. Next, we reduce the PDE model (1) to a more tractable model for control, using a projection Galerkin method or assumed mode approach, e.g. [10], [11].

To be able to apply the assumed mode approach, let us first apply the following one-to-one change of coordinates to the equation (1)

$$u(y,t) = w(y,t) + \frac{l(t) - y}{l(t)} f_1(t) + \frac{y}{l(t)} f_2(t) \quad (5)$$

One can easily see that this change of coordinates implies trivial boundary conditions

$$\begin{array}{c} w(0,t) = 0 \\ w(l(t),t) = 0 \end{array} \quad (6)$$

After some algebraic and integral manipulations, the PDE model (1) writes in the new coordinates as

$$\begin{array}{c} \rho\frac{\partial^2 w}{\partial t^2} + 2v(t)\rho\frac{\partial^2 w}{\partial y \partial t} + \left(\rho v^2 - T(y,t)\right)\frac{\partial^2 w}{\partial y^2} \\ +(G(t) + vk_{dp}\delta(x - l + l_{dp}))\frac{\partial w}{\partial y} \\ +(c_p + k_{dp}\delta(x - l + l_{dp}))\frac{\partial w}{\partial t} \\ = y\left(-\rho s_1(t) - c_p s_2(t)\right) - \rho f_1^{(2)} + s_4(t) \\ -\frac{\partial w}{\partial t}\left(\frac{l-y}{l} f_1(t) + \frac{y}{l} f_2(t)\right) k_{dp}\delta(x - l + l_{dp}) \\ -vk_{dp}\frac{f_2 - f_1}{l}\delta(x - l + l_{dp}) \end{array} \quad (7)$$

where $G(t) = \rho a(t) - \frac{\partial T}{\partial y} + c_p v(t)$, and the s_i variables are defined as

$$\begin{array}{l} s_1(t) = \frac{l l^{(2)} - 2\dot{l}^2}{l^3} f_1(t) + 2\frac{\dot{l}}{l^2}\dot{f}_1 \\ + \frac{(l^3 f_2^{(2)} - f_2 l^2 l^{(2)} + 2l\dot{l}^2 f_2 - 2l^2\dot{l}\dot{f}_2)}{l^4} - \frac{f_1^{(2)}}{l} \\ s_2(t) = \frac{\dot{l}}{l^2} f_1 - \frac{\dot{f}_1}{l} + \frac{\dot{f}_2}{l} - f_2\frac{\dot{l}}{l^2} \\ s_3(t) = \frac{f_2 - f_1}{l} \\ s_4(t) = -2v(t)\rho s_2(t) - G(t)s_3(t) - c_p\dot{f}_1(t) \end{array} \quad (8)$$

associated with the two-point boundary conditions

$$w(0,t) = 0, \quad w(l(t),t) = 0. \quad (9)$$

Now instead of dealing with the PDE (1) with non-zero boundary conditions, we can use the equivalent model, given by equation (7) associated with trivial boundary conditions (9).

Following the assumed-modes technique, the solution of the equation (7), (9) writes as

$$w(y,t) = \sum_{j=1}^{j=N} q_j(t)\phi_j(y,t), \quad N \in \mathbb{N} \quad (10)$$

where N is the number of bases (modes), included in the discretization, $\phi_j,\ j = 1,...,N$ are the discretization bases and $q_j,\ \ j = 1,..,N$ are the discretization coordinates. In order to simplify the analytic manipulation of the equations, the base functions are chosen to satisfy the following normalization constraints

$$\int_0^{l(t)} \phi_j^2(y,t)dy = 1,\ \int_0^{l(t)} \phi_i(y,t)\phi_j(y,t)dy = 0,\ \ \forall i \neq j \quad (11)$$

To further simplify the base functions, we define the normalized variable, e.g. [5], [3] $\xi(t) = \frac{y(t)}{l(t)}$, and the normalized base functions $\phi_j(y,t) = \frac{\psi_j(\xi)}{\sqrt{l(t)}},\ \ \ j = 1,...,N$. In these new coordinates the normalization constraints write as $\int_0^1 \psi_j^2(\xi)d\xi = 1,\ \int_0^1 \psi_i(\xi)\psi_j(\xi)d\xi = 0,\ \ \forall i \neq j$. After discretization of the PDE-based model (7), (8) and (9) (e.g. refer to [3]), we can write the reduced ODE-model based on N-modes as

$$M\ddot{q} + (C + \tilde{C}U)\dot{q} + (K + \tilde{K}U)q = F(t) + \tilde{F}(t)U,\ q \in \mathbb{R}^N \quad (12)$$

where

$$\begin{array}{l} U{=}k_{dp} \\ M_{ij}{=}\rho\delta_{ij} \\ C_{ij}{=}\rho l^{-1}\dot{l}\left(2\int_0^1(1{-}\xi)\psi_i(\xi)\psi_j'(\xi)d\xi{-}\delta_{ij}\right){+}c_p\delta_{ij} \\ \tilde{C}_{ij}{=}l^{-1}\psi_i(\frac{l-l_{dp}}{l})\psi_j(\frac{l-l_{dp}}{l}) \\ K_{ij}{=}\frac{1}{4}\rho l^{-2}\dot{l}^2\delta_{ij}{-}\rho l^{-2}\dot{l}^2\int_0^1(1{-}\xi)^2\psi_i'(\xi)\psi_j'(\xi)d\xi \\ {+}\rho l^{-1}(g{-}a(t))\int_0^1(1{-}\xi)\psi_i'(\xi)\psi_j'(\xi)d\xi{+}m_e l^{-2}(g{-}a(t))\int_0^1\psi_i'(\xi)\psi_j'(\xi)d\xi \\ {+}\rho(l^{-2}\dot{l}^2{-}l^{-1}\ddot{l})\left(0.5\delta_{ij}{-}\int_0^1(1{-}\xi)\psi_i(\xi)\psi_j'(\xi)d\xi\right){-} \\ c_p\dot{l}l^{-1}(\int_0^1\psi_i(\xi)\psi_j'(\xi)\xi d\xi{+}0.5\delta_{ij}){+}0.5M_{cs}gl^{-2}\int_0^1\psi_i'(\xi)\psi_j'(\xi)d\xi \\ \tilde{k}_{ij}{=}l^{-2}\dot{l}\left({-}\psi_j'(\frac{l-l_{dp}}{l})\psi_i(\frac{l-l_{dp}}{l})(\frac{l-l_{dp}}{l}){-}0.5\psi_i(\frac{l-l_{dp}}{l})\psi_j(\frac{l-l_{dp}}{l}) \right.\\ \left.{+}\psi_j'(\frac{l-l_{dp}}{l})\psi_i(\frac{l-l_{dp}}{l})\right) \\ F_i(t){=}{-}l\sqrt{l}(\rho s_1(t){+}c_p s_2(t))\int_0^1\psi_i(\xi)\xi d\xi \\ {+}\sqrt{l}\left(s_4(t){-}\rho f_1^{(2)}(t)\right)\int_0^1\psi_i(\xi)d\xi \\ \tilde{F}_i(t){=}\frac{\dot{f}_1}{\sqrt{l}}\psi_i(\frac{l-l_{dp}}{l}){+}\frac{l-l_{dp}}{\sqrt{l}}\psi_i(\frac{l-l_{dp}}{\sqrt{l}})(\dot{l}l^{-2}(f_1{-}f_2){+}l^{-1}(\dot{f}_2{-}\dot{f}_1)) \\ \delta_{ij} = \left\{\begin{array}{l} 0,\ i \neq j \\ 1,\ i = j \end{array}\right. \end{array} \quad (13)$$

where $i,j \in \{1,...,N\}$, and $s_k,\ k = 1,2,3,4$ are given in (8).

Remark 1: The model (12), (13) has been obtained for the general case of time-varying rope length $l(t)$, however, in this paper we only consider the case of stationary ropes $l = cte$, which is directly deduced from (12), (13), by setting $\dot{l} = \ddot{l} = 0,\ \forall t$. The case where the car is static and the control system

has to reject the ropes' oscillations is of interest in practical setting. Indeed, besides the case of commercial buildings at night (where the elevators are not in use), in many situations where the building is swaying due to external strong weather conditions, the elevators are stopped, for the security of the passengers. The control system is then used to damp out the ropes sway, to avoid the ropes from damaging the elevator system, and be still functional after the external disturbances have passed. [1]

III. MAIN RESULT: LYAPUNOV-BASED SEMI-ACTIVE DAMPER CONTROL

The first controller deals with the case where the building, hosting a stationary elevator (stopped at a given floor), sustains a brief (impulse-like) external disturbance. For example, an earthquake impulse with a sufficient force to make the top of the building oscillate, or a strong wind gust that happens over a short period of time, exciting the building structure and implying residual vibrations of the building even after the wind gust interruption. In these cases, the elevator ropes will vibrate, starting from a non-zero initial conditions, due to the impulse-like external disturbances (i.e., happening over a short time interval), and this case correspond to the model (12), (13) with non-zero initial conditions and zero external disturbances. We can now state the following theorem.

Theorem 1: Consider the rope dynamics (12), (13), with non-zero initial conditions, with no external disturbances, i.e., $f_1(t) = f_2(t) = 0, \forall t$, then the feedback control

$$U(z) = u_{max} \frac{\dot{q}^T \tilde{C} \dot{q}}{\sqrt{1 + (\dot{q}^T \tilde{C} \dot{q})^2}} \tag{14}$$

where $z = (q^T, \dot{q}^T)^T$, implies that $q(t) \to 0,\ for\ t \to 0$, furthermore $|U| \leq u_{max},\ \forall t$, and $|U|$ deceases with the decrease of $\dot{q}^T \tilde{C} \dot{q}$.

Proof: We define the control Lyapunov function as

$$V(z) = \frac{1}{2}\dot{q}^T(t) M \dot{q}(t) + \frac{1}{2} q^T(t) K(t) q(t) \tag{15}$$

where $z = (q^T, \dot{q}^T)^T$.

First we compute the derivative of the Lyapunov function along the dynamics (12), without disturbances, i.e., $F(t) = 0,\ \tilde{F}(t) = 0 \forall t$

$$\begin{aligned} \dot{V}(z) &= \dot{q}^T(-C\dot{q} - \tilde{C}U\dot{q} - Kq) + q^T K \dot{q} \\ &= -\dot{q}^T C \dot{q} - \dot{q}^T \tilde{C} \dot{q} U \end{aligned} \tag{16}$$

[1] Of course, there are also practical cases were the cars are in motion and an external disturbance occurs. These cases correspond to a time-varying rope length, which we have also studied, however, due to space limitation, we could not include all the results in this paper. The case of time-varying rope length will be presented in another report.

Next, using U defined in (14), we have

$$\dot{V}(z) \leq -u_{max} \frac{(\dot{q}^T \tilde{C} \dot{q})^2}{\sqrt{1 + (\dot{q}^T \tilde{C} \dot{q})^2}} \tag{17}$$

Using LaSalle theorem, e.g. [12], and the fact that $\tilde{C}$ is symmetric positive definite we can conclude that the states of the closed-loop dynamics converge to the set $S = \{z = (q^T, \dot{q}^T)^T \in \mathbb{R}^{2N},\ \ s.t.\ \dot{q} = 0\ \}$. Next, we analyze the closed-loop dynamics: Since the boundedness of V implies boundedness of $\dot{q},\ q$ and by equation (12), boundedness of $\ddot{q}$. Boundedness of $\dot{q},\ \ddot{q}$ implies the uniform continuity of $q,\ \dot{q}$, which again by (12), implies the uniform continuity of $\ddot{q}$. Next, since $\dot{q} \to 0$, and using Barbalat's Lemma, e.g. [12], we conclude that $\ddot{q} \to 0$, and by invertibility of the stiffness matrix $K + \beta U$ we conclude that $q \to 0$. Finally, the fact that V is a radially unbounded function, ensures that the equilibrium point $(q, \dot{q}) = (0, 0)$ is globally asymptotically stable. Furthermore the fact that $|U| \leq u_{max}$, and the decrease of $|U|$ as function of $\dot{q}^T \tilde{C} \dot{q}$ is deduced from equation (14), since $\frac{\dot{q}^T \tilde{C} \dot{q}}{\sqrt{1 + (\dot{q}^T \tilde{C} \dot{q})^2}} \leq 1$ and $\frac{1}{\sqrt{1 + (\dot{q}^T \tilde{C} \dot{q})^2}} \leq 1$. ■

Remark 2: It is clear from equation (16) that the trivial choice of a constant positive damping control U, will also imply a convergence of the sway dynamics to zero. However, the controller (14) has the advantage to require less control energy comparatively to a constant damping, since by construction of the control law (14), the stabilizing damping force decreases together with the decrease of the sway.

The controller U given by (14) does not take into account the disturbance $F(t)$ explicitly. Next, we present a controller which deals with the case of a static elevator in a building under sustained external disturbances, e.g. sustained wind forces on commercial buildings at night where the cars are static. In this case $F(t) \neq 0,\ \tilde{F}(t) \neq 0$ over a non-zero time interval, and satisfy the following assumption.

Assumption 1: The time varying disturbance functions $f_1,\ f_2$ are such that, the functions $F(t),\ \tilde{F}(t)$ are bounded, i.e. $\exists (F_{max},\ \ \tilde{F}_{max}),\ \ s.t.\ \ |F(t)| \leq F_{max},\ \ |\tilde{F}(t)| \leq \tilde{F}_{max},\ \forall t$.

Theorem 2: Consider the rope dynamics (12), (13), under non-zero external disturbances, i.e., $f_1(t) \neq 0,\ f_2(t) \neq 0$ satisfying Assumption 1, with the feedback control

$$\begin{aligned} &U(z) = u_{max_p} \frac{\dot{q}^T \tilde{C} \dot{q}}{\sqrt{1+(\dot{q}^T \tilde{C} \dot{q})^2}} \\ &+ \dot{q}^T \tilde{C} \dot{q} \frac{v_{1max}(\tilde{F}_{max}|\dot{q}| v_{1max} + \tilde{F}_{max}|\dot{q}| u_{max_p})}{\sqrt{1+(\dot{q}^T \tilde{C} \dot{q})^2 (\tilde{F}_{max}|\dot{q}| v_{1max} + \tilde{F}_{max}|\dot{q}| u_{max_p})^2}} \\ &+ \dot{q}^T \tilde{C} \dot{q} \frac{v_{2max}(|\dot{q}| F_{max} + v_{2max} \tilde{F}_{max}|\dot{q}|)}{\sqrt{1+(\dot{q}^T \tilde{C} \dot{q})^2 (|\dot{q}| F_{max} + v_{2max} \tilde{F}_{max}|\dot{q}|)^2}} \end{aligned} \tag{18}$$

where $u_{max_p},\ v_{1max},\ v_{2max} > 0$, are chosen s.t. $(u_{max_p} + v_{1max} + v_{2max}) \leq u_{max}$ and $z = (q^T, \dot{q}^T)^T$. Then, if we define the two invariant sets:

$$\begin{aligned} &S_1 = \{(q^T, \dot{q}^T)^T \in \mathbb{R}^{2N},\ s.t. \\ &\frac{(\dot{q}^T \tilde{C} \dot{q})^2}{\sqrt{1+(\dot{q}^T \tilde{C} \dot{q})^2 (\tilde{F}_{max}|\dot{q}| v_{1max} + \tilde{F}_{max}|\dot{q}| u_{max_p})^2}} \\ &\leq \frac{1}{v_{1max}}\} \end{aligned}$$

, and

$$S_2 = \{(q^T, \dot{q}^T)^T \in \mathbb{R}^{2N},\ s.t. \ \frac{(\dot{q}^T\tilde{C}\dot{q})^2}{\sqrt{1+(\dot{q}^T\tilde{C}\dot{q})^2(|\dot{q}|F_{max}+v_{2max}\tilde{F}_{max}|\dot{q}|)^2}} \leq \frac{1}{v_{2max}}\}$$

, the controller (18) ensures that the state vector z converges to the the invariant set S_1 or S_2, and that $|U| \leq u_{max},\ \forall t$.

Proof: Let us consider again the Lyapunov function (15). Its derivative along the dynamics (12), with non-zero disturbance, i.e. $F(t) \neq 0,\ \tilde{F} \neq 0$, writes as

$$\begin{array}{l} \dot{V}(x) = \dot{q}^T(-C\dot{q} - \tilde{C}U\dot{q} - Kq + \tilde{F}U) \\ +q^TK\dot{q} + \dot{q}^TF(t) \\ = -\dot{q}^TC\dot{q} - \dot{q}^T\tilde{C}U\dot{q} + \dot{q}^T\tilde{F}U + \dot{q}^TF(t) \\ \leq -\dot{q}^T\tilde{C}U\dot{q} + \dot{q}^T\tilde{F}U + \dot{q}^TF(t) \end{array} \tag{19}$$

Now, we use the concept of Lyapunov reconstruction, e.g. [13], we write the control

$$U = u_{nom_p} + v_1 + v_2 \tag{20}$$

where, u_{nom_p} is the nominal controller, given by (14) with $u_{max} = u_{max_p}$, designed for the case where $F(t) = \tilde{F}(t) = 0,\ \forall t$. The remaining terms $v_1,\ v_2$ are added to compensate for the effect of the disturbances $\tilde{F}$ and F, respectively. We design v_1 and v_2 in two steps:

- First step: We assume that $\tilde{F} \neq 0,\ F(t) = 0,\ \forall t$, and design v_1 to compensate for the effect of $\tilde{F}$. In this case $U = u_{nom_p} + v_1$.

In this case, the Lyapunov function derivative is bounded as as follows

$$\begin{array}{l} \dot{V}(x) \leq -\dot{q}^T\tilde{C}(u_{nom_p}+v_1)\dot{q} + \dot{q}^T\tilde{F}(u_{nom_p}+v_1) \\ \leq -\dot{q}^T\tilde{C}v_1\dot{q} + \dot{q}^T\tilde{F}(u_{nom_p}+v_1) \\ \leq -\dot{q}^T\tilde{C}\dot{q}v_1 + \dot{q}^T\tilde{F}u_{nom_p} + \dot{q}^T\tilde{F}v_1 \end{array}$$

which under Assumption 1, gives

$$\leq -\dot{q}^T\tilde{C}\dot{q}v_1 + |\dot{q}|\tilde{F}_{max}u_{max_p} + |\dot{q}|\tilde{F}_{max}v_{1max}$$

Now, if we define (to simplify the notations), the term

$$T_1 = +|\dot{q}|\tilde{F}_{max}u_{max_p} + |\dot{q}|\tilde{F}_{max}v_{1max}$$

and if we choose the controller

$$v_1 = \frac{v_{1max}T_1\dot{q}^T\tilde{C}\dot{q}}{\sqrt{1+T_1^2(\dot{q}^T\tilde{C}\dot{q})^2}}$$

This leads to the following Lyapunov function derivative upper bound

$$\leq T_1\big(1 - \frac{v_{1max}(\dot{q}^T\tilde{C}\dot{q})^2}{\sqrt{1+T_1^2(\dot{q}^T\tilde{C}\dot{q})^2}}\big) = B_1$$

- Second step: We assume that $\tilde{F}(t) \neq 0,\ F(t) \neq 0$, and design v_2 to compensate for F. In this case we write the total control as $U = u_{nom_p} + v_1 + v_2$. Now the upper-bound of the Lyapunov function derivative (along the total control) writes as

$$\begin{array}{l} \leq B_1 - \dot{q}^T\tilde{C}\dot{q}v_2 + \dot{q}^T\tilde{F}v_2 + \dot{q}^TF \\ \leq B_1 - \dot{q}^T\tilde{C}\dot{q}v_2 + |\dot{q}|\tilde{F}_{max}v_{2max} + |\dot{q}|F_{max} \end{array}$$

Next, if we define the term

$$T_2 = |\dot{q}|\tilde{F}_{max}v_{2max} + |\dot{q}|F_{max}$$

and if we choose the controller

$$v_2 = \frac{v_{2max}T_2\dot{q}^T\tilde{C}\dot{q}}{\sqrt{1+T_2^2(\dot{q}^T\tilde{C}\dot{q})^2}}$$

This leads to the following Lyapunov function derivative upper-bound

$$\begin{array}{l} \leq B_1 + T_2\big(1 - \frac{v_{2max}(\dot{q}^T\tilde{C}\dot{q})^2}{\sqrt{1+T_2^2(\dot{q}^T\tilde{C}\dot{q})^2}}\big) \\ \leq B_1 + B_2 \end{array}$$

where $B_2 = T_2\big(1 - \frac{v_{2max}(\dot{q}^T\tilde{C}\dot{q})^2}{\sqrt{1+T_2^2(\dot{q}^T\tilde{C}\dot{q})^2}}\big)$.

By choosing $v_{1max},\ v_{2max}$ high enough the two terms $B_1,\ B_2$ will be made negative. We can then analyze two cases:

1- First, the trajectories keep decreasing until they reach the invariant set

$$S_1 = \{(q^T, \dot{q}^T)^T \in \mathbb{R}^{2N},\ s.t.\ B_1 \geq 0\}$$

Then, the trajectories can either keep decreasing (if $|B_2| > B_1$) until they enter the invariant set

$$S_2 = \{(q^T, \dot{q}^T)^T \in \mathbb{R}^{2N},\ s.t.\ B_2 \geq 0\}$$

or they get stuck at S_1.

2- Second, the trajectories decrease until they reach the invariant set S_2 first, and stay there, or keep decreasing until they reach the invariant set S_1.

In both cases, the trajectories will end up in either S_1 or S_2. Finally, the fact that $|U| \leq u_{max}$ is obtained by construction of the three terms, since from (14), we can write $|u_{nom_p}| \leq u_{max_p}$ and by construction $|v_1| \leq v_{1max}$, $|v_2| \leq v_{2max}$, which leads to $|U| \leq u_{max_p} + v_{1max} + v_{2max} \leq u_{max}$. ■

Remark 3: We want to underline here the fact that, contrary to the previous case of impulse disturbances, in this case of sustainable external disturbances, the use of a simple passive damper, i.e. $U = cte$, might actually destabilize the system. Indeed, by examining equation (19), we can see that if the U is constant the positive term $+\dot{q}^T\tilde{F}U$, which is due the external disturbance, could overtake the damping negative term $-\dot{q}^T\tilde{C}U\dot{q}$, leading to instability of the system.

Remark 4: The controllers (14), (18) are state feedbacks based on $q,\ \dot{q}$, these states can be easily computed from the sway measurements at N given positions $y(1), ..., y(N)$, via equation (10). The sway $w(y,t)$ can be measured by laser displacement sensors placed at the positions $y(i),\ i = 1, 2, ...N$, along the rope, e.g.[14], subsequently q can be computed by simple algebraic inversion of (10), and $\dot{q}$ can be obtained by direct numerical differentiation of q.

Parameters	Definitions	Values
n	Number of ropes	$8[-]$
m_e	Mass of the car	$3500[kg]$
ρ	Main rope linear mass density	$2.11[kg/m]$
l	Rope maximum length	$390[m]$
H	Building height	$402.8[m]$
c_p	Damping coefficient	$0.0315[N.sec/m]$

TABLE I

NUMERICAL VALUES OF THE MECHANICAL PARAMETERS

IV. NUMERICAL EXAMPLE

In this section we present some numerical results obtained on the example presented in [1]. The case of an elevator system with the mechanical parameters summarized on Table I has been considered for the tests presented hereafter. We write the controllers based on the model (12), (13) with one mode, but we test them on a model with two modes. The fact is that one mode is enough since when comparing the solution of the PDE (7) to the discrete model (12) the higher modes shown to be negligible, and a discrete model with one mode showed a very good match with the PDE model, but to make the simulation tests more realistic we chose to test the controllers on a two modes model. Furthermore, to make the simulation tests more challenging we added a random white noise to the states fed back to the controller (equivalent to about ± 1 cm of error on the rope sway measurement from which the states are computed, see Remark 1), and we filtered the control signal with a first order filter with a cut frequency of 10 hz and a delay term of 5 sampling times, to simulate actuator dynamics and delays due to signal transmission and computation time. First, to validate Theorem 1, we present the results obtained by applying the controller (14), to the model (12), (13), with non-zero initial conditions $q(0) = 20,\ \dot{q}(0) = 5$, and zero external disturbances, i.e. $f_1(t) = f_2(t) = 0,\ \forall t$. In these first tests, to show the effect of the controller (14) alone, without the 'help' of the system's natural damping, we fix the damping coefficient to zero, i.e. $c_p = 0$. We apply the controller (14), with $u_{max} = 10^9$ $Nsec/m$. Figures 2, 3[2] show the rope sway obtained at half rope-length $y = 195$ m with and without control. Without control the rope sway reaches a maximum value of about 1.5 m. With control we see clearly the expected damping effect of the controller, which reduces the sway amplitude by half. The corresponding control force is depicted on Figures 4, 5. We see that, as expected from the theoretical analysis of Theorem 1, the control force remains bounded by a maximum value of $40kNsec/m$, which is easily realizable by existing semi-active damper, e.g. magnotorheological damper. Furthermore, as proven in Theorem 1, the control amplitude decreases with the decrease of the sway.

[2]The figures' zoom is included for the reader to have a better idea about the signals shape.

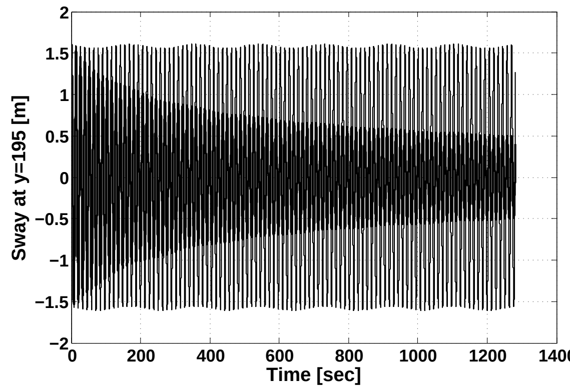

Fig. 2. Rope sway at $y = 195$ m: No control (thin line)- With controller (14) (bold line)

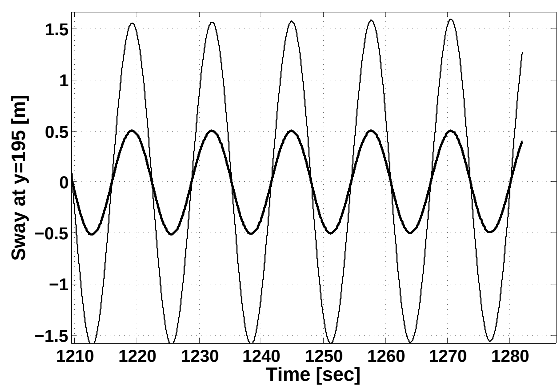

Fig. 3. Zoom of rope sway at $y = 195$ m: No control (thin line)- With controller (14) (bold line)

Next, we consider the model (12), (13) with no-zero disturbance signals: $f_1(t) = 0.2sin(2\pi.0.08t)$, and f_2 being deduced from f_1 via equation (3). We underline that we have purposely selected the disturbance frequency to be equal to the first resonance frequency of the rope, to simulate the 'worst-case scenario'. In this case we apply the controller (18), with the parameters $u_{max_p} = 10^9$ $Nsec/m,\ v_{1max} = v_{2max} = 10^5$ $Nsec/m,\ F_{max} = \tilde{F}_{max} = 1$. We show on Figures 6, 7 the sway signal in the uncontrolled and the controlled case. We see that the sway steady state maximum amplitude is reduced from 8.4 m in the uncontrolled case to 2.4 m with control. The noisy (due to the simulated measurements noise) bounded and continuous control signals are reported on Figures 8, 9.

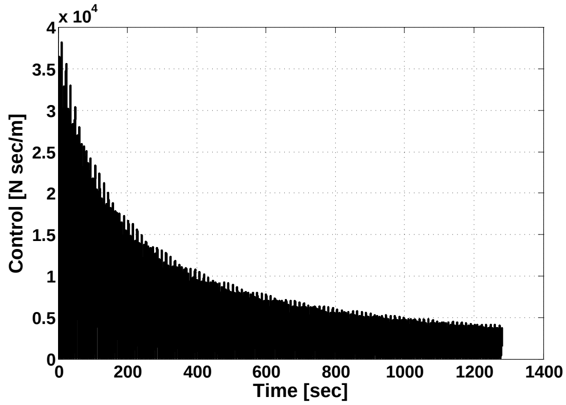

Fig. 4. Output of controller (14)

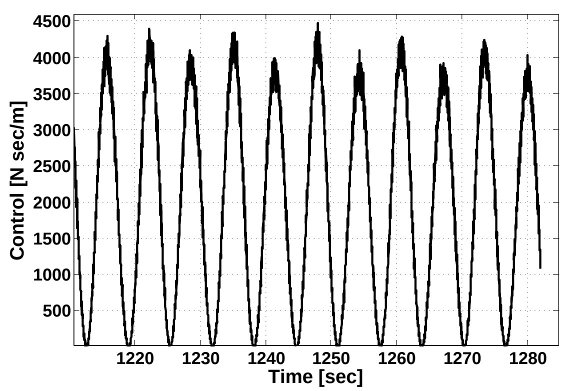

Fig. 5. Output of controller (14)- Zoom

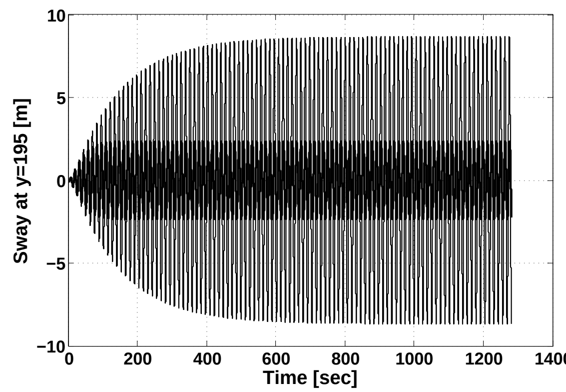

Fig. 6. Rope sway at $y = 195\ m$: No control (thin line)- With controller (18) (bold line)

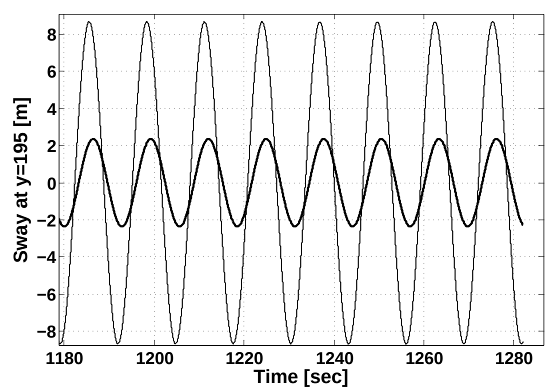

Fig. 7. Zoom of rope sway at $y = 195\ m$: No control (thin line)- With controller (18) (bold line)

V. Conclusion

In this paper we have studied the problem of semi active control of elevator rope sway dynamics occurring due to external force disturbances acting on the elevator system. We have considered the case of a static car, i.e. constant rope length and have proposed two nonlinear controllers based on Lyapunov theory. We have presented the stability analysis of these controllers and shown their efficiency on a numerical example. The semi-active stabilization problems related to time-varying rope lengths, i.e. moving car, will be presented in a future report.

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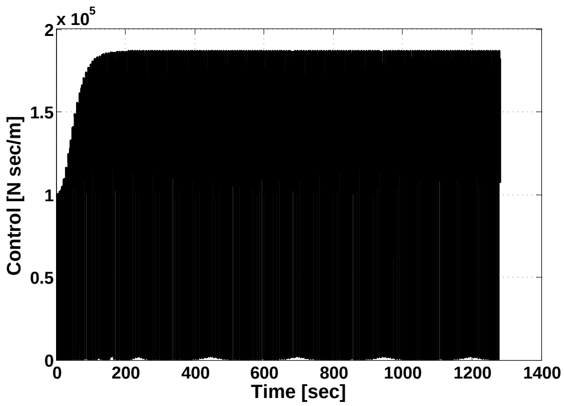

Fig. 8. Output of controller (18)

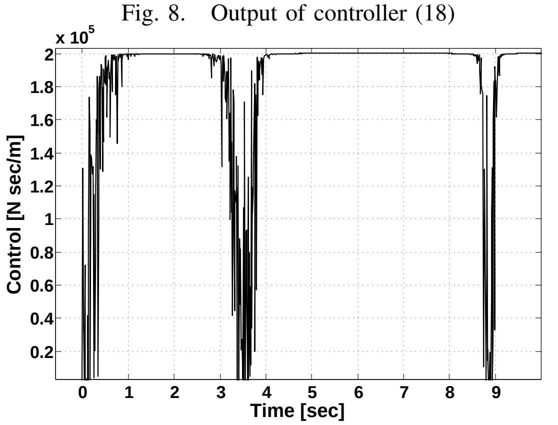

Fig. 9. Output of controller (18)- Zoom

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